# Astronomy 210

*Section 1– MWF 1500-1550*

*134 Astronomy Building*

This Class (Lecture 8):

Theory of Planetary Motion

Next Class:

The Two-Body Problem

***Planetarium Observing***

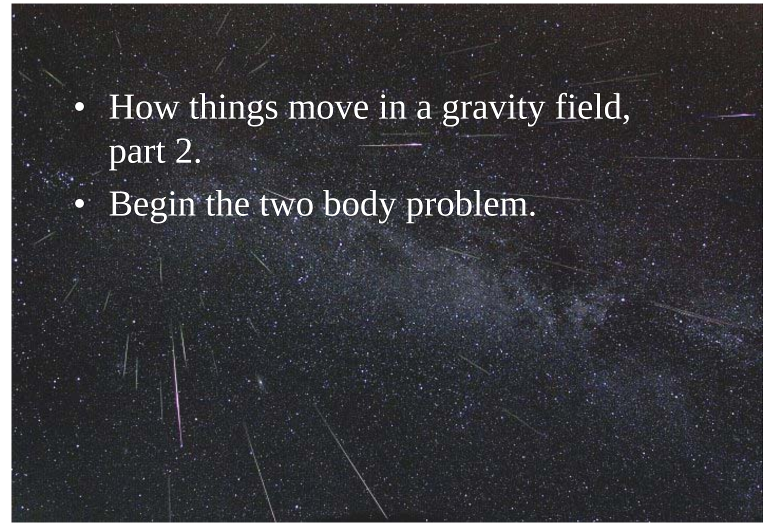
***Stardial Observing***

***Kaler lecture at Planetarium, Fri @ 7pm*** -> 1/6th HW grade (due Feb 11th)

Music: *Space Dog* – Tori Amos



# Outline



- How things move in a gravity field, part 2.
- Begin the two body problem.



# Result

- Now we know that the orbiting planets are just perpetually falling bodies. This includes the shuttle, satellites, etc.
- “Weightlessness” is just like falling. There is gravity on the shuttle, but as one is in freefall it is not noticeable.
- Kepler had thought briefly about this, but he decided he needed forces along the direction of the velocity, not perpendicular to it.
- So Newton realized that like an apple falling from a tree or a really big tree, the moon must have a force toward the Earth.
- Newton did not discover gravity, but he realized that it was universal.



# Applications

- We now have two tools in our box:
  - Newton's laws of motion: how things move with, without forces applied
  - Newton's gravity: the net force on solar system bodies
- Now, we need to out the two together: how do bodies move under the inverse square force of gravity.

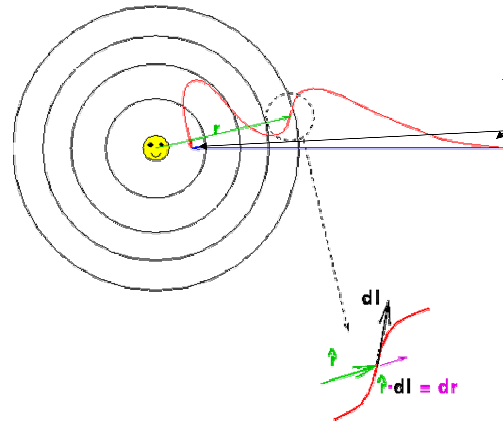
# Kinetic Energy

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left|\dot{\vec{r}}^{\,2}\right|$$

http://www.nap.edu/html/oneuniverse/images/energy_12_small.jpg
http://www.army-technology.com/projects/thaad/thaad3.html



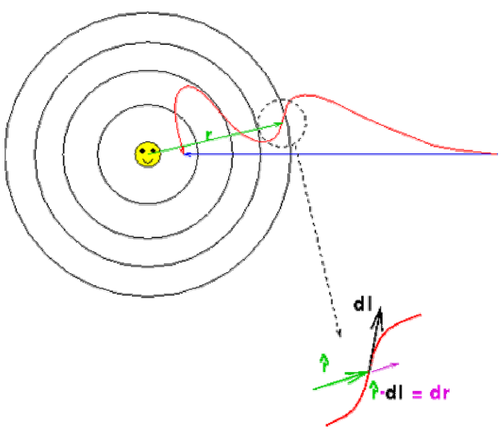
# Potential Energy of Gravity




$$\Delta U = -\int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$



# Potential Energy of Gravity

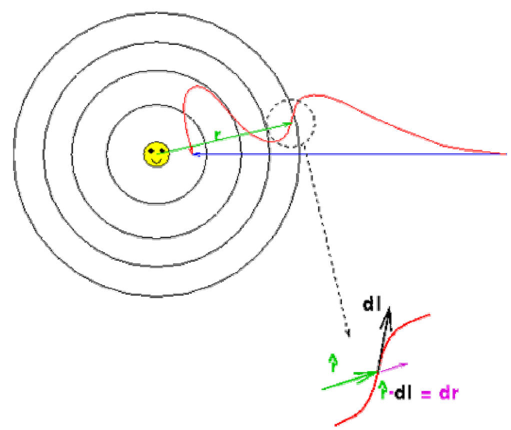



$$\Delta U = \int_{r_i}^{r_f} G\frac{Mm}{r^2}dr$$

Define potential energy at $r_f = \infty$ to be 0



# Potential Energy of Gravity




$$\boxed{U = -G\frac{Mm}{r}}$$

# Potential Energy of Gravity

$$\boxed{U = -G\frac{Mm}{r}}$$

Negative potential energy?

What's up with that?

It really implies that the object is bound by the gravitational field. In order to escape the pull of gravity, it has to be given that much energy.

The only physically meaningful quantity is relative changes in potential energy.



# Total Energy in Gravity

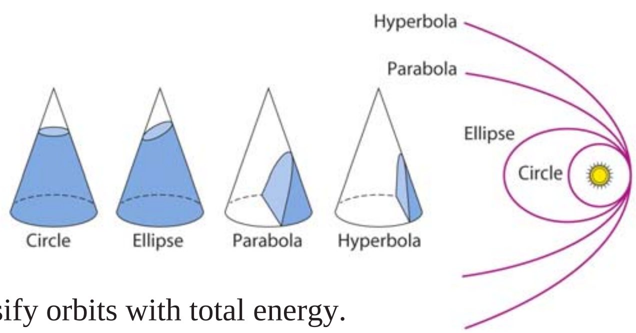
- TE = KE + PE

$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

**Conserved!!!!**



# Orbits




Can classify orbits with total energy.

Can write a circle or ellipse in polar coordinates as:

$$r(\theta) = \frac{(1-e^2)a}{1+e\cos\theta}$$



# Orbits

Consider a circle with e=0

Recall that $v_c^{\ 2} = G\frac{M}{r}$

so $KE = \frac{1}{2}mv^2$

# *Orbits*

Consider a circle with e=0

But $PE = -G\dfrac{Mm}{r}$

Then, KE = -PE/2

And TE = KE + PE = PE/2 **TE < 0!!**



# *Spooky Orbits*

**What?**
**Negative total energy?**

Is that due to spooky antimatter?

- No it means that you have to supply energy to system to break it apart.
- When particles at rest, far away, KE = PE = 0. To get to this situation, have to input energy to an orbiting system.
- Since energy is conserved, can't spontaneously get this energy, so system is **⇒bound!**
- Note: KE = -1/2PE generally true for gravitating systems in equilibrium: "virial theorem"



# *General Orbits*

Ellipse: 0 < e < 1

- It turns out that TE depends only on *a*, not on *e*
- From conservation of energy can see:

$$TE = -G\frac{Mm}{r} + \frac{1}{2}mv^2 = -G\frac{Mm}{2a}$$

**< 0!!**

**⇒ Bound!**



# *General Orbits*

Ellipse: 0 < e < 1

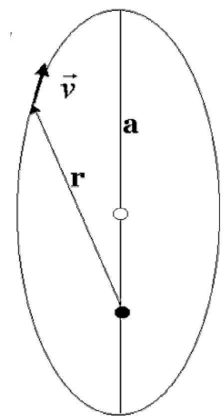
- It turns out that TE depends only on *a*, not on *e*
- From conservation of energy can see:

$$TE = -G\frac{Mm}{r} + \frac{1}{2}mv^2 = -G\frac{Mm}{2a}$$

# General Orbits

Ellipse: $0 < e < 1$

- It turns out that TE depends only on *a*, not on *e*
- From conservation of energy can see:




$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

This is called the "Vis Viva" equation.
The most handy orbital equation.

http://groups.msn.com/AndysUsingScienceFictionForEducationPage/visvivaequation.msnw



# General Orbits

Parabola: $e = 1$

- Can write as: $$r = \frac{p}{1+\cos\theta}$$
- Where p is distance at closest approach.
- Just unbound system (can barely escape to r=∞)
  E = 0 or KE = -PE

For Earth. $V_{escape}$ = 11km/s = 25000 mph.

$$v_{escape} = \sqrt{2G\frac{M}{r}}$$



# General Orbits

Hyperbola: $e > 1$

- Can write as: $$r(\theta) = \frac{(1-e^2)a}{1+e\cos\theta}$$
- Completely unbound system (escapes to r=∞)
  E > 0
- That implies that v > 0 at r=∞



# Life is a 2 Body Problem

- So far, I have cheated a little… or simplified
- Not the planets, nor the Sun is nailed down.
- From Newton's third, the Sun must move!
  (but more mass so acceleration is smaller)

How do binary stars orbit?

We have to understand a 2 body system.

## *Consider a Box*

- A box in space with mass away from any gravity field.
- How would it move?
- What if the box had many pieces inside of it, how would it move?



## *Consider a Box*

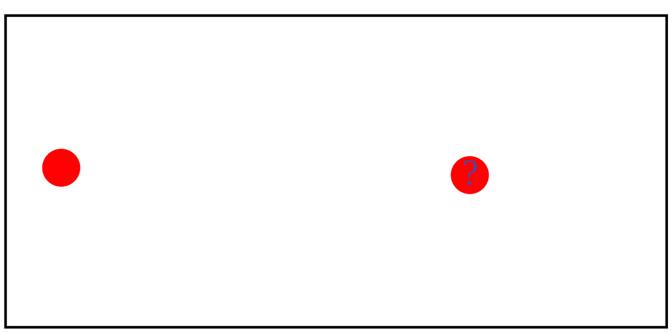
- Let our box have 2 objects with masses $m_1$ and $m_2$
- How would it move?

**No matter what the pieces do, the box still acts as a free body.**
**⇒Constant velocity.**



## *Consider a Box*

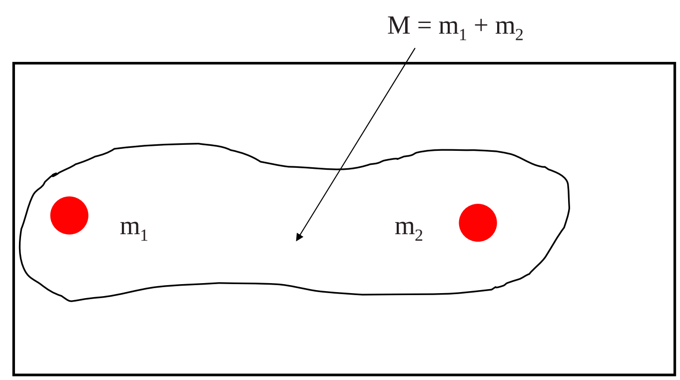
- If you are moving with the box, at the same velocity:




Can we say anything about where the second mass has to be?



## *Reduce Problem*

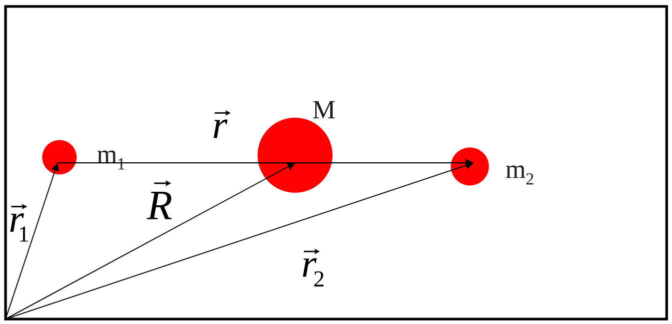



If we treat as a single object of mass M with no external forces (only gravity between them), then how does the box move?

# Center of Mass (COM)

$\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$





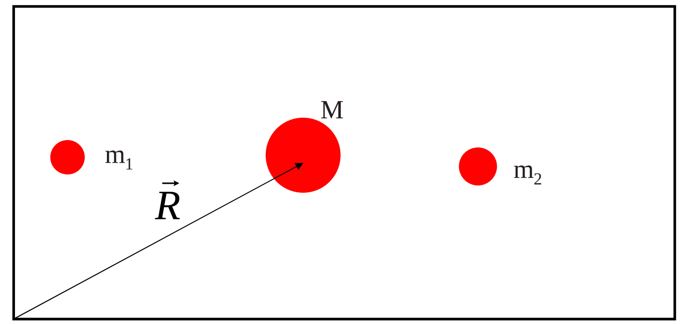

# Center of Mass (COM)

With no external forces, can show that

$$\ddot{\vec{R}} = 0 \Rightarrow \dot{\vec{R}} = \text{constant}$$

Can pick inertia frame where

$$\dot{\vec{R}} = 0$$





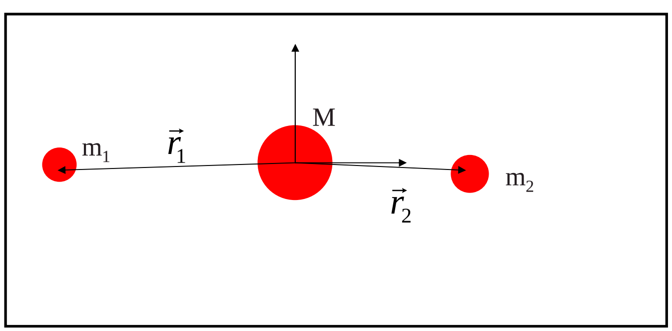

# Center of Mass (COM)

With no external forces, can show that

$$\ddot{\vec{R}} = 0 \Rightarrow \dot{\vec{R}} = \text{constant}$$

Can pick reference frame where

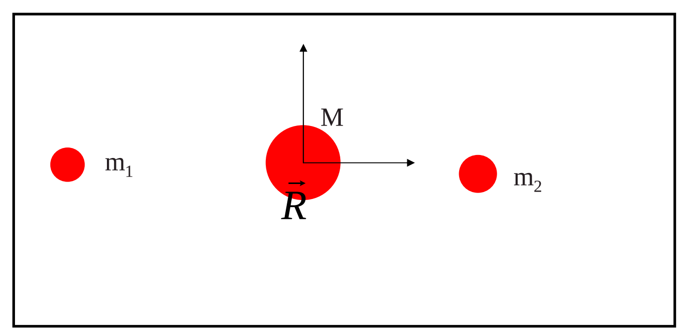
$$\vec{R} = 0$$






# Center of Mass (COM)

What we are really doing is trading one pair of vectors (positions of each mass) for a different pair (COM and relative separation) which are more convenient and natural...

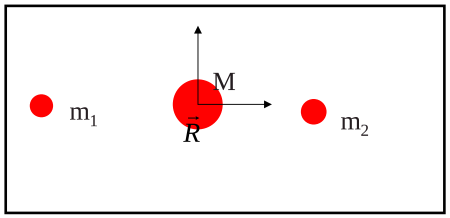


# COM

- So can think of two objects orbiting the center of mass $M = m_1 + m_2$
- Each star has its own orbit with the same eccentricity and a different, but related semimajor axis.
- The stars are always opposite each other about the center-of-mass. Both stars will reach peristron (closest approach) at same time.
- If m1>> m2, then we're back to

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \approx \vec{r}_1$$

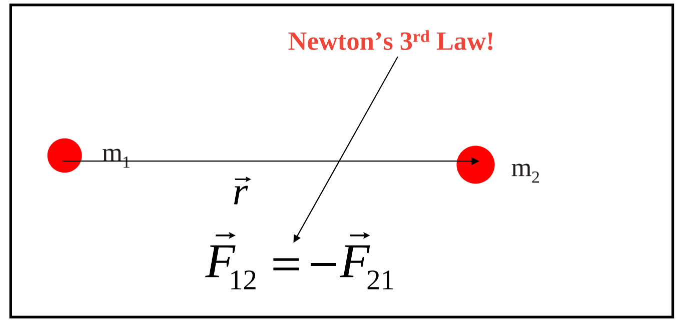
$$M = m_1 + m_2 \approx m_1$$






# Force in the Box

$$\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\hat{r} = G\frac{m_1 m_2}{r^3}\vec{r} \qquad \vec{F}_{21} = -G\frac{m_1 m_2}{r^3}\hat{r}$$

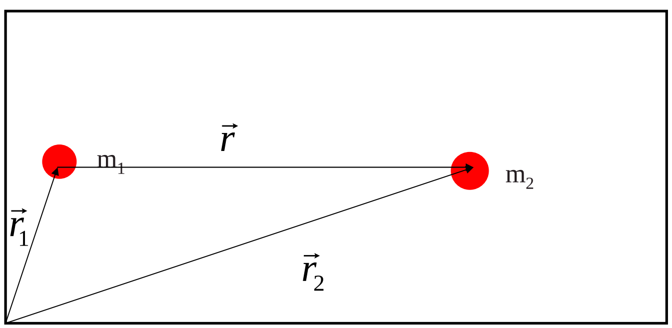





# Reduced Mass

$$m_1\ddot{\vec{r}}_1 = \vec{F}_{12}$$

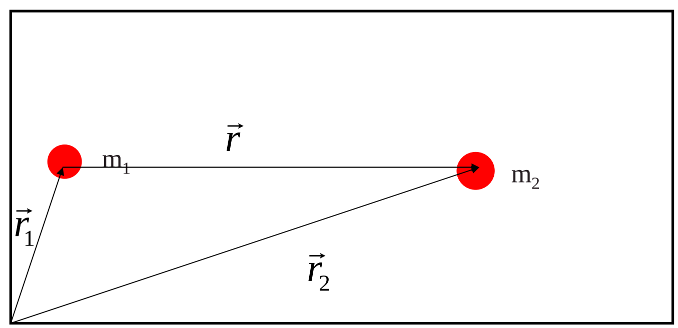
$$m_2\ddot{\vec{r}}_2 = \vec{F}_{21}$$





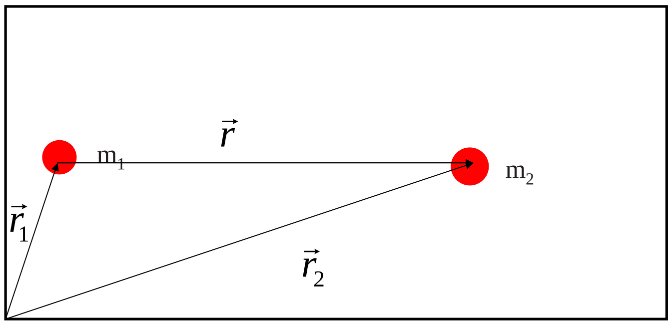

# Reduced Mass

$$= \vec{F}_{21}\left(\frac{m_1 + m_2}{m_1 m_2}\right)$$



# Reduced Mass

$$define\ \mu = \frac{m_1 m_2}{m_1 + m_2}\quad reduced\ mass \qquad then\ \ \mu\ddot{\vec{r}} = \vec{F}_{21}$$

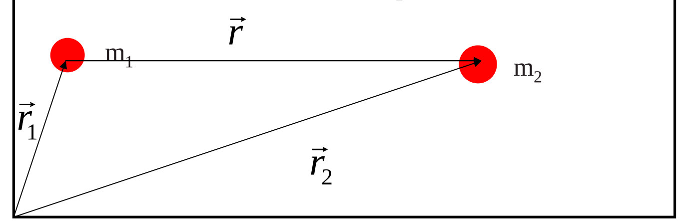





# Reduced Mass

$\mu\ddot{\vec{r}} = \vec{F}_{21}$ Identical to simplified case, but now mass is modified.

$$note: if\ \ m_2 << m_1\ then\ \mu \approx \frac{m_1 m_2}{m_1} = m_2 \ \text{ or } \ddot{\mathrm{F}}_{21} = m_2\ddot{\vec{r}}$$






# Reduced Mass

$\mu\ddot{\vec{r}} = \vec{F}_{21}$

- System can be thought of a *test particle* of reduced mass that is orbiting a mass M = $m_1 + m_2$
- Different reference frames.