Astronomy 210

Section 1– MWF 1500-1550

134 Astronomy Building

This Class (Lecture 7):

Gravitation

Next Class:

Theory of Planetary Motion

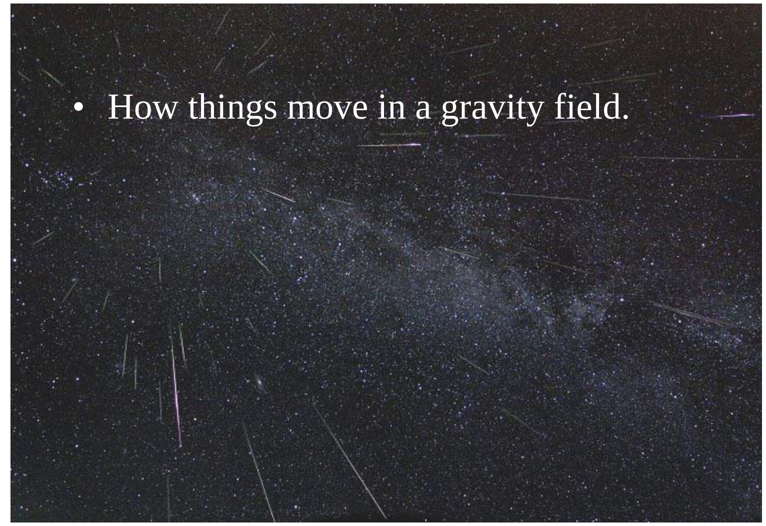
HW #2 Due Friday!

Missed 2nd planetarium date. (only 5 left), including tonight

Stardial Observing report is available

Music: *Earthbound* – Darrin Drda

Outline



- How things move in a gravity field.

Angular Momentum

What is $\vec{r} \times \vec{F}$?

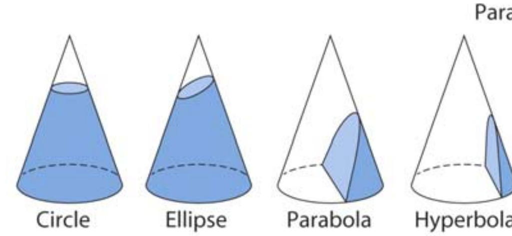
$$\vec{L} = \vec{r} \times m\vec{v}$$

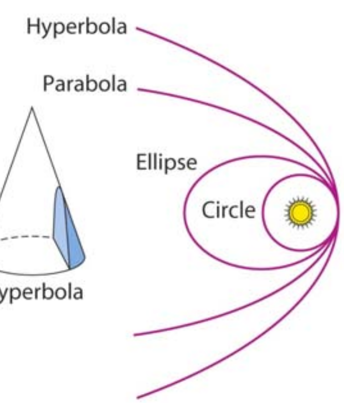
What about gravity guaranteed that dL/dt =0?

Conserved!!!!

☺ *Kepler's 1st Law by Newton*

- Newton also found more options that satisfied his universal law of gravity
- For any attractive inverse square law force, can show that the orbits are conic sections with the other body at one focus.





Circular Orbit: Special Case

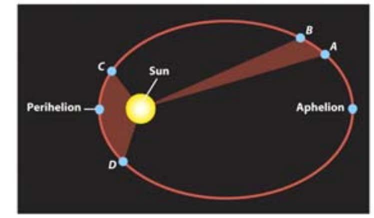
$$F = ma = \frac{m v_c^2}{r}$$

Example:

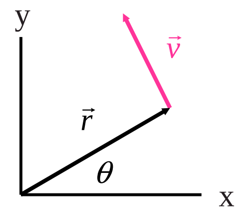
Circular speed at 1 AU from Sun is 3 x 10^4 m/s

Kepler's 2nd Law by Newton

Do orbits in Newton's formalism sweep out equal area in equal time?

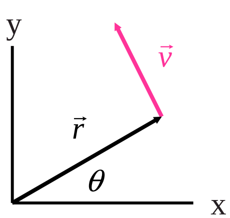


$$d\theta = \omega dt$$



Kepler's 2nd Law by Newton

$$dA = \frac{1}{2} r^2 \omega dt$$

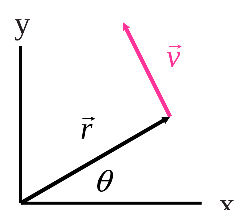


Kepler's 2nd Law by Newton

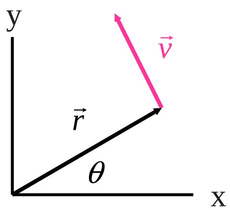
$$dA = \frac{1}{2} r^2 \omega dt$$

$$v = \omega r$$

$$dA = \frac{1}{2} r^2 \frac{v}{r} dt$$

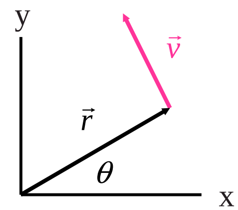


Kepler's 2nd Law by Newton



Kepler's 2nd Law by Newton

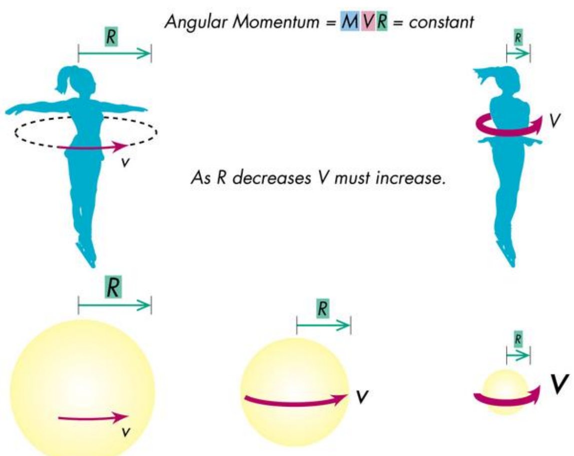
Then



$$\frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$$ **Constant!!**

Kepler's 2nd Law by Newton

- The product of mass, orbital velocity, and orbital radius is constant while in orbit.



Kepler's 3rd Law by Newton

Remember for circular motion

$$v_c = \sqrt{G\frac{M}{a}}$$

so

$$a^3 = \left(\frac{GM}{4\pi^2}\right)P^2$$

Kepler's 3rd Law by Newton

- Remember Kepler's 3rd Law

$$\boxed{P^2 = a^3}$$

- Newton's found a general version with his Law of Gravity

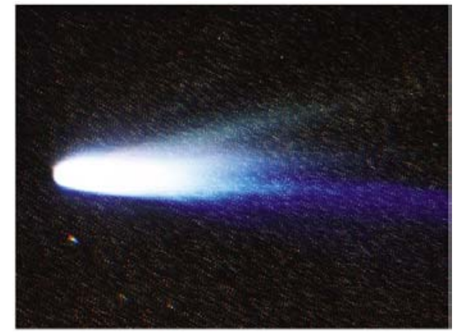
$$\boxed{P^2 = \underbrace{\left(\frac{GM}{4\pi^2}\right)}_{k} a^3}$$

(P in years, a in AUs, M in solar masses)

$\Rightarrow k = 1$

The Power of Newton's Laws

- An astronomer named Edmund Halley was intrigued that a comet appeared about every 76 years
  - Was it the same comet every time?
- Using Newton's Laws, Halley determined the comet's orbit around the Sun
- Predicted its return in 1758
  - It will be back in 2061!



Spaceflight

- Rockets operate by Newton's 3rd Law
  - Action: Gas expelled out the end of the rocket
  - Reaction: Rocket is pushed upward

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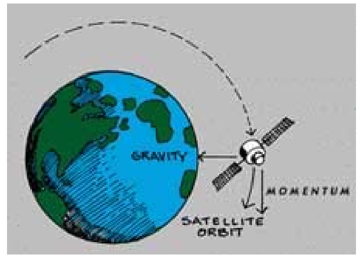
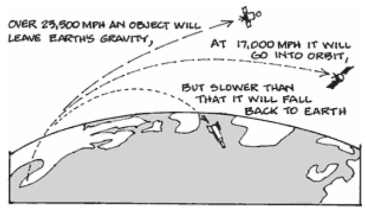
Orbiting the Earth

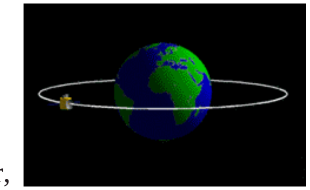
- In orbit, spacecraft is “falling around the Earth”
  - A spacecraft must have a velocity of 28,000 km/hr (17,000 mph) to stay in orbit
  - If its velocity exceeds 11 km/sec (25,000 mph), it can escape the Earth’s gravity





Geosynchronous Orbit

- At a distance of 42,200 km from the Earth's center, a satellite takes 24 hours to orbit the Earth
  - Equal to the Earth's rotation
  - If a satellite is above the Earth's equator, it will appear stationary from the ground
- This orbit is called *geosynchronous*
  - Useful for communication satellites
  - Receivers can point at one spot in the sky



Result

- Now we know that the orbiting planets are just perpetually falling bodies. This includes the shuttle, satellites, etc.
- “Weightlessness” is just like falling. There is gravity on the shuttle, but as one is in freefall it is not noticeable.
- Kepler had thought briefly about this, but he decided he needed forces along the direction of the velocity, not perpendicular to it.
- So Newton realized that like an apple falling from a tree or a really big tree, the moon must have a force toward the Earth.
- Newton did not discover gravity, but he realized that it was universal.

Applications

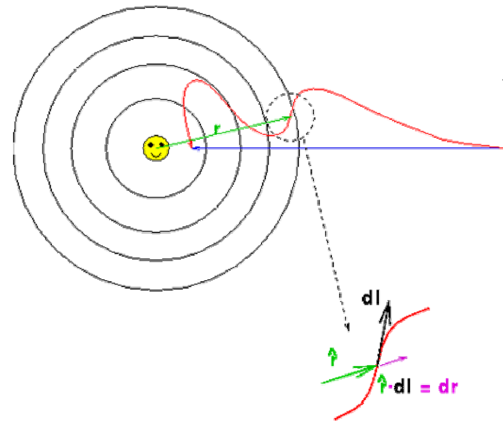
- We now have two tools in our box:
  - Newton's laws of motion: how things move with, without forces applied
  - Newton's gravity: the net force on solar system bodies
- Now, we need to out the two together: how do bodies move under the inverse square force of gravity.

Kinetic Energy

http://www.nap.edu/html/oneuniverse/images/energy_12_small.jpg

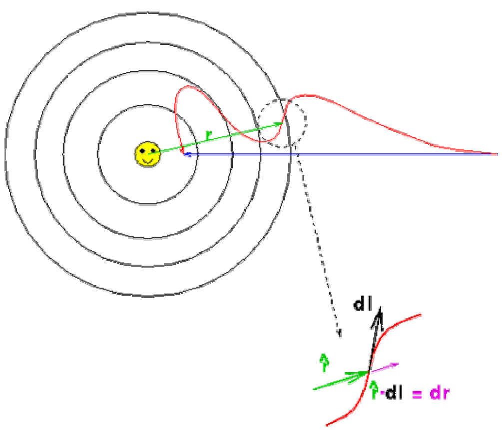
http://www.army-technology.com/projects/thaad/thaad3.html

Potential Energy of Gravity



$$\Delta U = -\int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

Potential Energy of Gravity



$$U = -G\frac{Mm}{r}$$

Potential Energy of Gravity

$$U = -G\frac{Mm}{r}$$

Negative potential energy?

What's up with that?

It really implies that the object is bound by the gravitational field. In order to escape the pull of gravity, it has to be given that much energy.

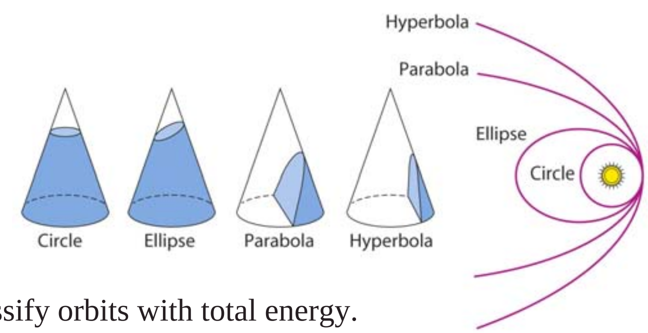
The only physically meaningful quantity is relative changes in potential energy.

Total Energy in Gravity

- TE = KE + PE

Conserved!!!!

Orbits



Can classify orbits with total energy.

Can write a circle or ellipse in polar coordinates as:

Orbits

Consider a circle with e=0

Orbits

Consider a circle with e=0

TE < 0!!

Orbits

What?
Negative total energy?

Is that due to spooky antimatter?

- No it means that you have to supply energy to system to break it apart.
- When particles at rest, far away, KE = PE = 0. To get to this situation, have to input energy to an orbiting system.
- Since energy is conserved, can't spontaneously get this energy, so system is **⇒bound!**
- Note: KE = -1/2PE generally true for gravitating systems in equilibrium: "virial theorem"

General Orbits

ellipse: 0 < e < 1

- It turns out that TE depends only on *a*, not on *e*
- From conservation of energy can see:

< 0!!

⇒ Bound!

General Orbits

ellipse: 0 < e < 1

- It turns out that TE depends only on *a*, not on *e*
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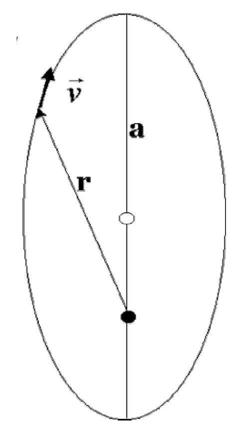
General Orbits

ellipse: 0 < e < 1

- It turns out that TE depends only on *a*, not on *e*
- From conservation of energy can see:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$



This is called the "Vis Viva" equation.
The most handy orbital equation.

http://groups.msn.com/AndysUsingScienceFictionForEducationPage/visvivaequation.msnw

General Orbits

parabola: e = 1

- Can write as:
- Where p is distance at closest approach.
- Just unbound system (can barely escape to r=∞)
  E = 0 or KE = -PE

For Earth. V_{escape} = 11km/s = 25000 mph.

$$\Rightarrow \qquad v_{escape} = \sqrt{2G\frac{M}{r}}$$

General Orbits

hyperbola: e > 1

- Can write as: $$r(\theta) = \frac{(1-e^2)a}{1+e\cos\theta}$$
- Completely unbound system (escapes to r=∞)
  E > 0
- That implies that v > 0 at r=∞