Astronomy 210 Spring 2005
Homework #9

Due in class: Friday, April 15.

Problems

1. (10 points) Sometime during the week, go outside on a clear night and find the constellation Orion. Find the altitude of Betelgeuse at 3 different times on the same night (e.g. 8pm. 11pm, and 1am or whenever you go to bed). Make sure to write down the time, date, altitude, and sky location (e.g. southern sky). Compare to your result from HW 3. Keep in mind that there has been a time change.

2. *The Solar Interior*

   (a) (3 points) We can estimate the pressure at the center of the Sun in the following way. We know that the Sun is in hydrostatic equilibrium. As shown in class, this means that at any distance r from the center, we have

   $$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

   where ρ is the density of the Sun at r, and $M(r)$ is to total mass of the Sun inside a radius r, so that $M(0) = 0$, while $M(R_\odot) = M\odot$. Let's simplify this differential equation into an algebraic one by assuming that the Sun has uniform density and pressure, which both go abruptly to zero at its surface $r_{surface} = R_\odot$. Then we can approximate $dPr/dr \approx \Delta P/\Delta r$, where the differences $\Delta r = r_{surface} - r_{center} = R_\odot$ and $\Delta P = P_{surface} - P_{center} = -P_{center}$. are taken between the surface and center. Also put $M(r) = M_\odot$ and $\rho(r) = \rho_{avg}$. In this approximation, find the central pressure $P_c = P_{center}$ of the Sun, and compare your answer to the atmospheric pressure on the Earth's surface.

   (b) (3 points) Now calculate the total number N of particles in the Sun. To get this, note that we know $M_\odot$, so that $M = Nm$, where m is the average mass of a particle. But we know that the Sun is mostly ionized H, so there are equal numbers of proton and electrons, with an average particle mass $m = (m_p + m_e)/2 \approx m_p/2$.

   (c) (3 points) Using the results from (a) and (b), as well as the ideal gas law, estimate the central temperature T_c of the Sun. How does this compare to the surface temperature $T_\odot$? Detailed models of the Sun's structure give $T_c = 1.6 \times 10^7$ K. Comment on the accuracy of our estimate.

3. As discussed in class, the Sun is powered by energy released in nuclear reactions. These reactions occur in chains which have the net effect of burning hydrogen to helium-4: $4p \rightarrow {}^4\mathrm{He} + 2\nu + 2e^+$. The production of each new ^{4}He nucleus liberates $E(4p \rightarrow {}^4\mathrm{He}) = 26$ MeV $=$ $2.6\ \times 10^7$ eV $= 4.2\ \times 10^{-12}$ J, and creates two neutrinos.

   (a) (3 points) Using charge conservation, deduce the electric charge of the neutrino from the first reaction in the $p-p$ chain, $p + p \rightarrow {}^2\mathrm{H} + \nu + e^+$. Note that matter at the Sun's core is fully ionized, so that ^{2}H here is a deuterium nucleus, not a deuterium atom.

(b) (3 points) Given the total rate of solar energy output (i.e., the luminosity) of the Sun, $L_\odot = 3.85 \times 10^{26}$ W, calculate the number of helium nuclei that must be created per second. Use this to calculate the number of neutrinos that are created per second; call this the neutrino production rate $\mathcal{R}_\nu$.

(c) (3 points) The flux F_ν of solar neutrinos at the earth is the number of neutrinos arriving from the Sun per unit area per second. Show that this flux (at Earth) is $F_\nu = \mathcal{R}_\nu / 4\pi a^2$, with $a = 1$ AU. Calculate F_ν in units of [neutrinos $\mathrm{m}^{-2}\ \mathrm{s}^{-1}$].

(d) (3 points) Estimate the area A of your palm. The number of neutrinos passing through your outstretched hand in one second (during the daytime) is $F_\nu A \Delta t$, with $\Delta t = 1$ s. Calculate this number.

(e) (3 points) Comment on how you expect the number from part 3d to change when it is night.

(f) (3 points) What basic fact about solar energy is confirmed by the detection of solar neutrinos?

(g) (3 points) Estimate the lifetime of the Sun by assuming that it burns all its hydrogen into helium and has the same luminosity throughout its life.

4. (15 points short essay question: typed) From your Ferris readings, discuss the Harvard computers that were used to evaluate stellar spectra and how that led to the H-R diagram.