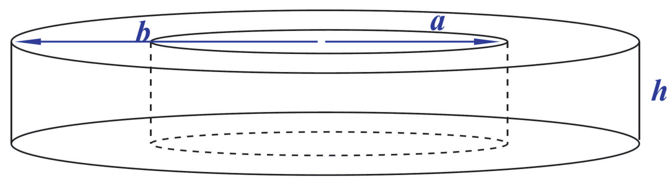
Astronomy 210 Spring 2005
Homework #8

Due in class: Friday, April 8.

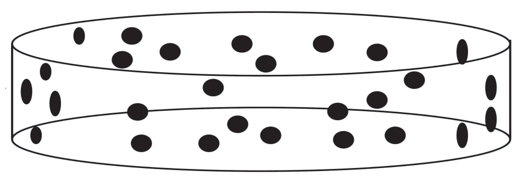
Problems

1. *Asteroid avoidance..* The number of asteroids with size above R = 1 km is about N = 10^5. Most of these objects have orbits with radii between a = 2.1 AU and b = 3.3 AU, with an average orbital inclination of i = 15° . Let us approximate the asteroid belt as a cylindrical shell, with inner and outer radii as given, and with a height which can be estimated as h = $2r_{avg} \sin i$, where r_{avg} is the average of the inner and outer radii of the shell.



   (a) (8 points) If the average asteroid has a radius R = 2 km, compute the total volume occupied by all asteroids. How does this compare to the total volume of the asteroid belt? Comment on the result. How "full" is the asteroid belt? How does this compare to Hollywood and video game representations?

   (b) (8 points) When we launch a satellite to the outer solar system, we want to avoid hitting asteroids. The key question is: how likely is a collision? One can view this as a "shooting darts" problem. Imagine for simplicity that all asteroids are at r_{avg}, but randomly distributed in height.



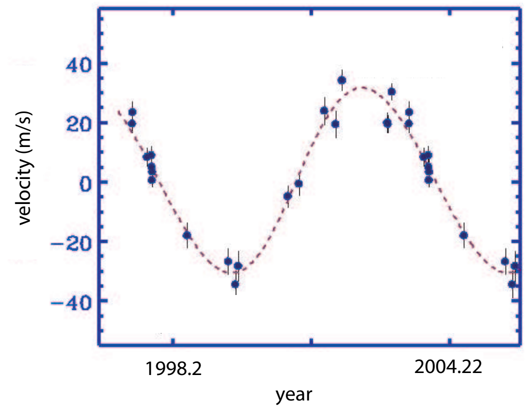
   This means that they are distributed over the rim of a cylinder of radius r_{avg} and height h, with a total surface area S_{tot} given in the usual way. (This is the projected area of the asteroid belt.) Let $S_{asteroid}$ be the total portion of this area covered by all the asteroids; this is just the sum of the cross sectional areas of each of the N asteroids. Our satellite (the "dart") must pass through a point somewhere on the total area (the "dartboard"), and we want to know how likely an asteroid (a "bullseye") will be there or not. The probability p of collision is defined so that p = 1 = 100% is a certain collision and p = 0 = 0% means no chance, so $0 < p < 1$ gives the odds in between. This probability depends on the net cross sectional area of the asteroids (the total size of all bullseyes). Namely, p is just given by the fraction of the total area which is actually covered by the asteroids, i.e.,

$$p = \frac{S_{asteroid}}{S_{total}}$$

   For example, if asteroids were to cover half the area, then we would find p = 0.50, and thus infer a 50% chance of collision when one passes through. Using the data above, calculate p as a number $\in$ (0,1), and as a percentage, and comment. How much of a risk does NASA take with each mission to the outer solar system?

2. *Extrasolar Planets.* We are in the midst of the pioneering days of discovering extrasolar planets. Consider a star of mass M with a planet of mass m $\ll$ M. Both the star and the planet orbit their common center of mass (COM), and both have an orbital period P about the center of mass that is equal to the period P of the planet around the star (the usual period from Kepler's law). The planet is detected not by direct observation, but by the periodic change the "wobble" it causes in the star's velocity.

   (a) (8 points) In the figure is the Doppler velocity data from a nearby star. The wobble is caused by an extrasolar planet. Calculate the orbital period of the planet in years.



   (b) (8 points) Newton's form of Kepler's law relates the period (which we can observe) to the semi-major axis a and the sum M+m of the masses. Knowing that the mass of the star observed is $M \approx 1M_{\odot}$, compute the semi-major axis; be careful of units, and show your work! Compare your result to the semi-major axis of Mercury.

   (c) (10 points) Now assume that the orbits are circular, so that the separation is always r = a. Using the result from part 4(b) of Homework 3, show that the orbital speed of the star about the COM is

$$v_* = \frac{m}{M+m} v = \frac{m}{M+m} \frac{2\pi}{P}$$

   The line-of-sight speed v_* is the Doppler data in the figure, and the magnitude of v_* is the amplitude of the sinusoidal pattern. Estimate the amplitude in this figure and compute the ratio m/M and then using $M \approx 1M_{\odot}$, compute the mass m of the planet in Jupiter masses.

   (d) (8 points) Find the speed $v_{\odot}$ of the Sun's motion around it's common center of mass with Jupiter. Comparing this with v_*, and $P_{Jupiter}$ with P, discuss what observational problems might make it harder to detect extrasolar planets similar to our own Jupiter than to detect this planet.