Astronomy 210 Spring 2005
Homework #7

Due in class: Friday, Mar. 18.

Problems

1. (7 points) The Earth's atmosphere affords a certain amount of protection from meteoroids. A meteoroid is slowed significantly by the atmosphere only if it encounters more than its own mass of air. Over a square centimeter of the Earth's surface, there are about 10^3 g = 1 kg of air. Making the approximation that meteoroids are spherical, what is the smallest meteoroid that will keep most of its incoming speed? Assume meteoroids have a density about the same a rocks, 3 g/cm^3.

2. In class we mentioned that Jupiter and Saturn radiate more energy than they take in from the Sun, and that this energy comes from gravitational contraction of the planet.

   (a) (4 points) The binding energy of a body of mass M and radius R is $E_G = -\alpha GM^2/R$. Assuming $\alpha = 0.4$, calculate the binding energy of Jupiter.

   (b) (6 points) Calculate the total energy received from Jupiter by the Sun (see earlier problem sets or lecture notes for an expression).

   (c) (4 points) Assuming that the mass of Jupiter is fixed but that the radius is gradually changing with time, take dE_G/dt and use this to relate the rate at which Jupiter's radius is changing to the rate at which gravitational binding energy is being released.

   (d) (4 points) Assuming that Jupiter's luminosity due to gravitational contraction is exactly equal to the amount of solar energy it intercepts, find the rate of change of Jupiter's radius. How long will it take Jupiter's radius to shrink by 10%?

3. (7 points) In the vacuum of space, ice sublimes (i.e., evaporates directly from a solid to water vapor) in a relatively short time at a temperature of 175 K or higher. Take the albedo of ice as A = 0.5, and calculate the distance from the Sun at which the equilibrium blackbody temperature of an icy body is just equal to this sublimation temperature. Where in the solar system can icy bodies exist?

4. In class, we calculated planetary temperatures as a function of distance d from the Sun, albedo A, and whether the planet has a thick atmosphere or not. Our simple first order treatment was idealized, and for example did not allow for greenhouse effect. We can make a simple but instructive estimate of the greenhouse effect as follows.

   (a) (3 points) First, find the magnitude of the greenhouse effect on Earth. In other words, we need to know what is our prediction for the average temperature without it. Use the expression derived in class, and the Earth's albedo, to predict the average surface temperature $T_\oplus$. How does this compare with the Earth's average surface temperature $T_{avg} = 15C$, that is, what is the temperature increase (due mostly to the greenhouse effect) above the predicted value?

(b) (7 points) Now model the greenhouse as follows. Imagine the greenhouse gasses in the Earth's atmosphere to be a single layer that is completely transparent to the visible wavelengths of the light received from the Sun, but completely opaque to the infrared radiation emitted by the surface of the Earth. Assume (1) that the top and bottom surface areas of the gas layer are each equal to the surface area of the planet, (2) that the entire layer is at the same temperature T_{gas}, and (3) that the layer radiates from both the top and the bottom (i.e. toward Earth and Space). Now, write an equation describing the energy balance for the gas layer, and another equation for energy balance at the Earth's surface. To do this, in each case identify the radiation power inputs and outputs, and then impose the condition of equilibrium by setting the input equal to the output.

(c) (3 points) Using the two equations you just found, show that the gas layer is at $T_{gas} = T_{\oplus}$.

(d) (5 points) Show that true surface temperature is $T_{surf} = 2^{1/4} T_{\oplus}$, and compute the numerical value of T_{surf}, in Kelvin and Celsius. How does this compare with T_{avg}, i.e., what do you calculate for the temperature increase due to greenhouse?