Astronomy 210 Spring 2005
Homework #6

Due in class: Friday, Mar. 11.

Problems

1. *A very bad day 65 million years ago.* As mentioned in class, the dinosaurs died suddenly, 65 million years ago. The cause for their death was uncertain until a large crater was found near the town of Chicxulub, in the Yucatan peninsula of Mexico. Analysis of material in the crater shows that it arose from an event 65 Myr ago, and thus coincided with– and probably caused– the extinction of the dinosaurs. Using the observed properties of the crater, and some physics, we can infer what it was like on the fateful day of impact.

   (a) (5 points) Assume for simplicity that the comet or asteroid is at rest at r = ∞. Show that when the object is at the radius R of Earth, its speed is just the escape velocity v_{esc} from Earth. Hint: consider the (conserved) energy of the impactor as it fell onto the Earth; you may ignore complications caused by the Earth's orbit and spin, and instead assume it to be stationary.

   (b) (5 points) The Chicxulub crater has a diameter D = 170 km. Roughly speaking, a impactor with diameter D leaves a crater of approximate diameter 20D. Use this relation and the density of rock, 3000 kg m^{-3}, to estimate the mass of the Chicxulub meteor. Assuming it had (Earth's) escape velocity upon impact, calculate its kinetic energy on impact, expressing your answer in Joules and in megatons of TNT (1 MTon = 4.2×10^{15} Joules). Compare to the global nuclear arsenal stores roughly 10^4 MTon.

   (c) (5 points) In the fraction of a second that impactor passed through the atmosphere, it would have been moving supersonically and so would have highly compressed the air around it, creating a shock wave. The temperature of the shocked gas can be crudely estimated by assuming that the impactor's disturbance gives gas particles typical random speeds comparable to the impactor's infall speed. If the average gas particle (i.e., a nitrogen atom) has a speed v_{rms} that is the same as the impactor, what temperature does this correspond to? Don't be surprised if this is very hot!

   (d) (5 points) Using the temperature you found in (c), calculate the brightness (i.e., the emitted flux F) from the air around the impactor, assuming blackbody emission. How does the brightness of the infalling object compare to the brightness of the Sun? This is what some poor T. Rex would have seen if it happened to look up on that very bad day.

2. (8 points) A rock sample is found to contain the radioactive potassium isotope ^{40}K ($t_{1/2} = 1.3 \times 10^9$ yr), and the stable argon isotope ^{40}Ar. If the rock solidified from a molten state, it was formed with no ^{40}Ar (which is gaseous), and only ^{40}K. If today the rock has $(^{40}K/^{40}Ar)_{today}$ = 3, what is the age of the rock? Hint: note that $(^{40}K)_{today}$ + $(^{40}Ar)_{today}$ = (^{40}K) , since the rock traps all of the ^{40}Ar decay products.

3. You have already considered the consequences of the fact that the Moon is receding from the Earth. Now let's derive the rate at which the Earth-Moon distance increases, given that the Earth's spin slows down due to the tidal interactions between the Earth and Moon. To do so, we'll take into account only the Earth's spin angular momentum, and the Moon's orbital angular momentum.

   (a) (5 Points) Show that, given a spin-down rate dP_E/dt, the Earth's spin angular momentum L_E changes according to

   $$dL_E/dt = -\frac{2}{5} M_E R_E^2 \frac{2\pi}{P_E^2}\, dP_E/dt$$

   where P_E is the Earth's spin period, and M_E and R_E are the Earth's mass and radius, respectively (which are constant!). Recall that the moment of inertia of a sphere is (2/5)MR^2.

   (b) (5 points) Assuming that the Moon remains on a circular orbit, show that its orbital angular momentum changes according to

   $$dL_M/dt = \frac{1}{2} M_M D_M \frac{2\pi}{P_M}\, dD_M/dt$$

   where P_M is the Moon's orbital period, and M_M and D_M are the Moon's mass and the Earth-Moon separation. Hint: you need to use the fact that P_M and D_M are related in a now-familiar way!

   (c) (3 points) Show that angular momentum conservation implies

   $$dD_M/dt = \frac{4}{5} \left(\frac{M_E}{M_M}\right) \left(\frac{R_E}{D_M}\right) \left(\frac{P_M}{P_E}\right) \left(\frac{R_E}{P_E}\right) dP_E/dt$$

   (d) (3 points) By how much does the Earth-Moon distance increase per year, if the Earth's spin slows down by dP_E/dt = 16 seconds per 1 million years?