Astronomy 210 Spring 2005
Homework #3
Due in Class: Friday, Feb. 11

Note: Your homework solutions should be legible and include all calculations, diagrams, and explanations. The TA is not responsible for deciphering unreadable or illegible problem sets! Also, homework is graded on the method of solution, not just the final answer; you may not get any credit if you just state the final answer!

1. Sometime during the week, go outside on a clear night and find the constellation Orion. Find the altitude of Betelgeuse at 3 different times on the same night (e.g. 8pm. 11pm, and 1am or whenever you go to bed). Make sure to write down the time, date, altitude, and sky location (e.g. southern sky). Comment on your results.

2. Consider a generalized gravitational force law, of the form , $\vec{F} = -k \frac{m1m2}{r^n} \hat{r}$ with n positive but otherwise arbitrary.

    a. Show that, for circular orbits with r = a = const, a force law of this kind leads to a generalized version of Kepler's third law, of the form $P_2 \alpha a^{\beta}$. Give an expression for β in terms of n. (3 points)

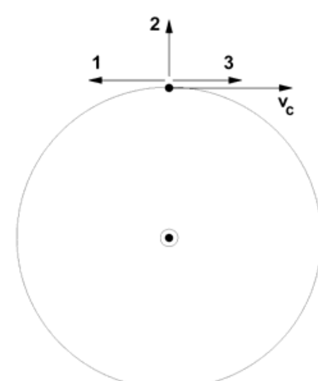
    b. Using your result from (a), show that the actual, observed form of Kepler's third law (i.e., β= 3) implies that n = 2. You will have thus shown that Kepler's third law demands that the gravitational force follow the inverse square law. (3 points)

    c. In the case of the generalized force law (with $n \neq 2$), would Kepler's second law change? Why? (2 points)

3. Cosmic trash disposal. Consider an object ("cosmic trash") of mass m which is in a circular orbit around the sun with speed v_c and radius a. To completely dispose of the waste, the options are either to drop it into the sun, or launch it out of the solar system.

    a. Without doing the calculation, which do you expect to be the cheapest waste removal and why? (2 points)

    b. To drop the trash into the sun requires that we remove the object's angular momentum, which costs us energy. Namely, we want to boost the material (i.e., change the velocity by firing rockets) so that it has a velocity v = 0 with respect to the sun; it then falls into the sun. To do this, the material must be boosted by a velocity $v_b = v_c$ in the opposite direction to its original orbit direction (vector 1 in the figure). Compute the boost energy required (in terms of m and v_c), where the boost is assumed to be instantaneous and thus all happens at distance r = a from the sun. (5 points)



    c. (b) To fling the material from the solar system also requires energy, as we want the object to be just unbound: KE +PE = 0. We can do this by boosting the object so that KE = -PE = $GM_{\odot}m/a$. If the boost (speed v_b) is aimed away from the sun (vector 2), this is perpendicular to the object's initial speed, and so the new speed with respect to the sun is $v^2 = v^2_b + v^2_c$. Compute the speed v_b needed to satisfy KE + PE = 0 in this case, and the boost energy needed (in terms of m and v_c). How does this compare with the energy in part (b)? (5 points)

d. Finally, we can eject the material from the solar system as in part (c), but now boost the material with speed v_b in the same direction as its initial orbit (vector 3). Then the object's new speed with respect to the sun is $v = v_b + v_c$. Compute the speed v_b needed to satisfy KE + PE = 0 in this case, and the boost energy needed (in terms of m and v_c). How does this compare with the energies in parts (a) and (b)? Which of these is the cheapest means of astronomical waste disposal? (5 points)

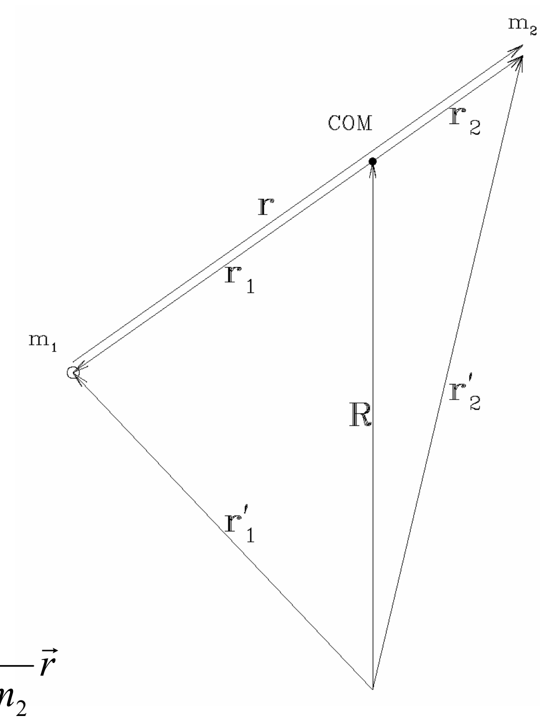
e. Did you guess right? Explain why or why not. (1 point)

4. Remember that the vector to the center of mass of two bodies m_1 and m_2 is

$$\vec{R} = \frac{m_1 r_1 + m_2 \vec{r}_2}{m_1 + m_2} \qquad (1)$$

and that the vector connecting the two bodies is $\vec{r} = \vec{r}_2' - \vec{r}_1'$ Also recall that the reduced mass is

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$



Assume that the only force acting on the masses is the gravitational attraction between them. Hint: you life will be easier if you treat the vectors as objects rather than break them down into components.

a) By the differentiation of eq (1) show that

$$\ddot{\vec{R}} = 0$$

b) Show that

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{r} \quad \text{and} \quad \vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{r}$$

c) Show that $r_1/r_2 = m_2/m_1$, where r_1 and r_2 are the magnitudes of the vectors.

d) Finally, consider the case of circular orbits. In Kepler's laws, the radius $a = r_1 + r_2$. Using this, calculate the Sun's distance to the Earth-Sun center of mass. How does this compare to the size of the Sun?