Astronomy 210 Spring 2005
Homework #11

Due in class: Wednesday, May 2.

Problems

1. (12 points) *Black Holes and Tidal Forces.* As punishment for yet another violation of the Prime Directive, the *USS Enterprise* is directed to actually do something of scientific use and is sent on a mission to decide whether a newly discovered compact object of mass M = 1.5 $M_\odot$ is a neutron star or a black hole. Mr. Spock (or Mr. Data, or T'Pol, as you see fit) object: "Captain, by my calculations, the odds are 356,137,289 to 1 that we will survive this encounter."

   Is our science officer correct? For our purposes, let's assume that the *Enterprise* is basically a steel rod of length $R = 500$ m, cross section $A = 100$ m^2, and mass $m = 10^7$ kg (if you are more familiar with the *Enterprise*'s technical specifications than I am, feel free to improve these numbers). Steel breaks when an applied force exceeds the breaking strength (that is, the "stress," or stretching force per unit area) of about 2×10^8 N/m^2. Estimate the distance d from the compact object at which the *Enterprise*'s hull will be torn apart by the tidal field. Compare with the radius of a neutron star and the Schwarzschild radius of the compact object. Is it possible to safely perform this mission? Hint: be mindful that what is dangerous to our heroes is not the average gravitational force on their spacecraft, but rather the tidal forces which describe the difference in gravitational force along the length of the ship.

2. *Stellar Radii.* A key physical parameter of a star is its radius. While we usually cannot measure this directly (i.e., geometrically) for most stars, we can infer it by realizing that (most) stars are basically blackbody emitters.

   (a) (5 points) Find an expression for a star's radius R given the star's luminosity L and surface temperature T, and assuming the star to be a blackbody emitter. It also useful and convenient to compare other stars to the Sun. Find an expression for $R/R_\odot$ given the star's luminosity and temperature, and $L_\odot$ and $T_\odot$.

   (b) (3 points) Now let's apply the expressions you have found. Consider the supergiant star Betelgeuse in the constellation Orion. Given the data in Appendix D you can find $L/L_\odot$ (you may want to use the information from HW10 on the Sun's magnitude). From the star's spectral type, and Appendix E, you can find its temperature T. (This is called "effective temperature" T_e in the Tables). Using these data, and the results from (a), compute $R/R_\odot$ for Betelgeuse. Comment on the result, and the appropriateness of the classification of such stars as "supergiants."

   (c) (3 points) Same as part (b), but for α Centauri (a.k.a. Rigel Kentaurus), which is a main sequence star. How does the radius compare to that of the Sun, also a main sequence star?

   (d) (3 points) The bright star Sirius (the "dog star") has a companion which is so dim that it is invisible to the naked eye. This companion is called Sirius B (and the dog star is then officially Sirius A). Sirius B has $L = 3 \times 10^{-3} L_\odot$ and T = 29,500 K. Comment on the result, and the appropriateness of the classification of such stars as “white dwarfs.”

(e) (3 points) The luminosity and surface temperature of the Sun when it undergoes helium flash will be about 2000 $L_\odot$ and 3000 K. At that time, what will be the Sun’s angular diameter in the sky as seen from the Earth?

3. (10 points) *White dwarf radius*. The size of a white dwarf (the ash left over after a normal star has burned its hydrogen into helium or other matter) is set by hydrostatic equilibrium– the balance between gravity and the pressure of the “degenerate” electrons in the white dwarf. The Pauli exclusion principle (or the Heisenberg uncertainty principle) states that at any point within the white dwarf, the average momentum p of an electron is related to its average separation x from its neighbor by $xp \simeq \hbar$, where $\hbar = h/2\pi$. If the electrons are non-relativistic, the energy of an average electron is $E_e = p^2/2m_e = \hbar^2/2m_e x^2$. Very roughly, for the whole star, hydrostatic equilibrium amounts to an equality $GM^2/R = N_e E_e$ between the gravitational potential energy GM^2/R and total kinetic energy of the degenerate electrons (where N_e is the number of electrons). The white dwarf’s volume is set by $N_e x^3 \simeq R^3$; it’s mass is set by $M \simeq N m_p$, where $N = 2N_e$ is the total number of neutrons and protons in the white dwarf, and m_p is the mass of the proton or neutron.

   Show that the white dwarf radius is given by

$$R = \frac{\hbar^2}{2^{8/3} G m_p^2 m_e} \left(\frac{M}{m_p} \right)^{-1/3} \tag{1}$$

   (Hint: impose hydrostatic equilibrium, and then eliminate x and N to express your answer in terms of the variables M and R.) Find the radius R of a $M = 1.05 M_\odot$ white dwarf. Compare your answer to the observed radius of Sirius B ($R = 0.0084 R_\odot$). How well does this model work?

4. (11 points) Pick a topic form your recent readings that was the most interesting to you and discuss. Relate it back to class discussions when possible. Type this $\approx$ 1 page.