Astronomy 210 Spring 2005
Homework #10

Due in class: Friday, April 22.

Problems

1. *Seeing ourselves as others see us.* The Sun has an apparent visual magnitude of $m_V = V = -26.75$. Refer to Appendix D and E of Carroll & Ostlie to answer the following:

   (a) (4 points) Calculate the apparent magnitude of the Sun at the distance of α Centauri (1.3 pc). Which is more luminous, the Sun or α Cen (primary)?

   (b) (4 points) If α Cen had the Sun's luminosity, would it be ranked among the 20 brightest stars in the sky, and if so, where?

   (c) (4 points) How far can the eye see? The human eye can see a star, under ideal conditions, as dim as an *apparent* visual magnitude $m_{V,max} = V_{max} = 6\ mag$. Use this to find the maximum distance d_{max}, in parsecs, from which your eye can see (a) the Sun, (b) an O5 supergiant, and (c) a main sequence M8 star. Compare these with the distance to the nearest star, and comment. Finally, comment on any possible bias that might affect the distribution of stars visible to the naked eye.

2. *Colors vs. temperature.* Here we will work out the relationship between $B-V$ color $= m_B - m_V$ and a star's effective temperature T. The star's spectrum is denoted F_λ, which is the measured energy flux per unit area per unit time *per unit wavelength.* The total flux from the star (energy per unit time per unit area) is then $\int_0^\infty d\lambda F_\lambda$. We will assume that the star's spectrum is proportional to

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda kT)} - 1}, \tag{1}$$

   which is the blackbody spectrum. Here $h = 6.62\times10^{-27}$, $c = 2.998\times10^{10}$, $k = 1.381\times10^{-16}$, in cgs units and λ is expressed in centimeters.

   Each photometric band is characterized by a filter function $\phi_B(\lambda)$ or $\phi_V(\lambda)$, which measures what fraction of the light is transmitted as a function of wavelength. Near the center of the band transmission is near 100%. The B band is centered at about 4500Å and is about 500Å wide ($\Delta\lambda_B = 500$Å), while the V band is centered at about 6000Å and is about 1000Å wide ($\Delta\lambda_V = 1000$Å). Note that 1Å $= 10^{-8}$cm.

   The observed flux in each band, which we will write F_B or F_V, is for example

$$F_B = \int d\lambda F_\lambda \phi_B(\lambda). \tag{2}$$

   Using the blackbody assumption,

$$F_B = \int d\lambda B_\lambda \phi_B(\lambda) \times const., \tag{3}$$

   where the constant is related to the star's radius and distance.

We will approximate this integral as

$$F_B \simeq \Delta\lambda_B B_\lambda(4500) \times const., \tag{4}$$

and similarly for F_V:

$$F_V \simeq \Delta\lambda_V B_\lambda(6000) \times const. \tag{5}$$

(a) (4 points) Based on the formulae above and the definition of apparent magnitude, write down a formula for the $B-V$ color of a star. Your formula will contain an undetermined constant.

(b) (4 points) Does the $B-V$ depend on the distance to the star?

(c) (5 points) Colors are calibrated according to a standard star. The star Vega, which is an A0 star with an effective temperature of 10^4K, has $B-V \approx 0$. Use this to set the undetermined constant from part (a).

(d) (5 points) What is the $B-V$ color of a $G2$ star, with an effective temperature of 5800K? What is the $B-V$ color of a $B0$ star with temperature $29,700$K?

3. *Stellar Evolution.* Go to the stellar evolution web page at

   `http://rainman.astro.uiuc.edu/ddr/stellar/intermediate.html`.

   Begin by evolving a set of 4 stars with solar metallicity and masses $1,2,3$ and $4M_\odot$. You can do this by typing `1 2 3 4` in the first box, then hitting submit. The evolution will take a couple of minutes, after which you will be able to see a link to a data file and a movie.

   Click in to the data file, which has 9 columns: time, logarithm base 10 of the temperature in Kelvins for star 1 (Log(T1)), logarithm base ten of the luminosity of star 1 in solar units (Log(L1)), then Log(T2), Log(L2), etc.

   (a) (4 points) By how much does the luminosity of a solar mass star change between zero age and 4.5 Gyr?

   (b) (4 points) What will the luminosity of the sun be 1 Gyr from now?

   (c) (4 points) Do the four stars obey the relationship $L \sim M^4$ discussed in class?

   (d) (4 points) What is the approximate lifetime of a $4M_\odot$ star (the time before the temperature drops precipitously)?

   (e) (4 points) Evolve a new model with only $1.5M_\odot$ star. What is the "Hertzsprung Gap Time" (this is about the main sequence lifetime, and it is listed on the web page where the link to the movie and data file are located)? Repeat with the metallicity (second box on the main evolution page) set to 0.002, which is a tenth of solar metallicity. What is the Hertzsprung Gap Time now? Speculate on why the two stars have different main sequence lifetimes.

4. **2.5 Bonus points plus a valuable and glamorous prize for the best submission.** Come up with your own original mnemonic for the spectral types OBAFGKMLT, to replace the standard "O Be A Fine Girl/Guy Kiss Me." Prizeworthy entries will be shown in class.